II. SOLUTION BY T. M. BLAKSLEE, Ames, Iowa.

Given $b + a$ and $a + h$, b, a, and h being the base, altitude, and hypothenuse respectively. Draw two parallel rays, l and s at distance $a + h$. On l lay off $AD = b + a$. Draw DE making $\angle ADE = 45°$. The ray of DE contains the vertex B of the required triangle ACB. This vertex is also on the parabola having s as directrix and A as focus.

Having found B, draw BC perpendicular to l and join A and B.

Excellent solutions were also received from A. M. Harding, C. N. Schmall, H. C. Feemster, Levi S. Shively, Elmer Schuyler, W. T. Risley, and Francis C. Rust.

408. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Given a point A on a circle and a chord of the circle; to draw a chord through A so that it shall be bisected by the given chord.

SOLUTION BY A. M. HARDING, University of Arkansas.

Let C be the center of the circle. On CA as a diameter describe a circle. This circle will cut the chord, in general, in two points P_1, P_2. Join A to P_1 and A to P_2. Then these are the required chords. If A be on the minor arc of the circle there will be two solutions. If A be on the major arc there will be two solutions, one solution, or no solution according as d is less than, equal to, or greater than $r - c$, where d is the distance from A to the given chord, c is distance from center to the given chord and r is the radius of given circle.

Also solved by T. M. Blakslee, C. N. Schmall, A. H. Holmes, Levi S. Shively, Francis C. Rust, G. W. Hartwell, W. R. Lebold, H. C. Feemster, and Elmer Schuyler.

CALCULUS.

326. Proposed by C. N. SCHMALL, New York City.

Prove that

$$\int_0^\infty \frac{(\tan^{-1} ax)^2 - (\tan^{-1} bx)^2}{x}\,dx = \frac{\pi^2}{4}(\log a - \log b).$$

SOLUTION BY THE PROPOSER.

Take the integral,

$$u = \int_0^\infty [f(x) - f(kx)]\frac{dx}{x}. \tag{1}$$

Substitute for x the successive values kx, k^2x, k^3x, $\cdots$, k^nx.

We then have

$$u = \int_0^\infty [f(kx) - f(k^2 x)]\frac{dx}{x}, \tag{2}$$

$$u = \int_0^\infty [f(k^2 x) - f(k^3 x)]\frac{dx}{x}, \tag{3}$$

$$\cdots\cdots\cdots\cdots$$

$$u = \int_0^\infty [f(k^{n-1} x) - f(k^n x)]\frac{dx}{x}. \tag{n}$$

Adding these equations, we get

$$nu = \int_0^\infty [f(x) - f(k^n x)] \frac{dx}{x} = \int_0^\infty [f(x) - f(rx)] \frac{dx}{x},$$

where $r = k^n$.

Now, suppose n to become infinite, r remaining constant, then k will approach unity as a limit, and we have

$$\int_0^\infty [f(x) - f(rx)] \frac{dx}{x} = n \int_0^\infty [f(x) - f(r^{1/n} x)] \frac{dx}{x}.$$

The right member of this equation (in the limit) reduces to the indeterminate form 0/0. Differentiating in this form with respect to $1/n$, we get

$$-\int_0^\infty f'(x) \log r dx = \log r[f(0) - f(\infty)].$$

Now in the given integral, $f(x) = (\tan^{-1} ax)^2$. Hence,

$$f(0) = 0, \qquad f(\infty) = \tfrac{1}{4}\pi^2;$$

also,

$$r = \frac{b}{a}.$$

Hence the integral is equal to $\frac{1}{4}\pi^2 (\log a - \log b)$.

This problem may be readily solved by Frullani's Theorem, Williamson's *Integral Calculus*, page 155. Professor J. Scheffer solved the problem by means of this Theorem, but failed to notice that in the above problem $\phi(ax) = [\tan^{-1} ax]^2$ instead of $\tan^{-1} ax$. Ed. F.

327. Proposed by RICHARD P. LOCHNER, Philadelphia, Pa.

A hound is at the middle point of the side of a square field and a fox is at an adjacent corner. How far will the hound run to catch the fox if the fox runs on the perimeter of the field and the hound runs directly towards the fox at all times, the hound running n times as fast as the fox. Where will the race end?

No solution of this problem has been received.

328. Proposed by M. E. GRABER, Tiffin, Ohio.

Prove that

$$\frac{E_m}{\pi/2m} \int_0^{\pi/2m} \left[\sin a + \sin\left(\frac{\pi}{m} + a\right) + \cdots + \sin\left(\pi \frac{m-1}{m} + a\right) da\right] = \frac{2mE_m}{\pi}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.

From trigonometry we have

$$\sin a + \sin\left(\frac{\pi}{m} + a\right) + \cdots + \sin\left(\pi \frac{m-1}{m} + a\right) = \frac{\sin\left(a + \dfrac{m-1}{2m} \cdot \pi\right)}{\sin \dfrac{\pi}{2m}}.$$

Hence the given integral takes the form